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Asymptotic parallel-in-time (APINT) development

Duration: 1 year. Researcher: Dr Jemma Shipton

Outline: Parallel-in-time (PINT) algorithms target the problem of lack of work at very large core counts, by operating on a 4-dimensional domain, instead of a 3-dimensional one. They operate by dividing the integration time period into segments and, starting from an initial guess for fields at the start of each time segment, computing the evolution of those fields, with processors working on each time segment in parallel. The process is iterated by combining the result with an approximate solution, produced by a fast "coarse propagator" to give new field values at the start of each time segmant which can now be evolved again using the full model. The process converges to give a physical solution that is continuous across the whole integration period; the discontinuities between the time segments converge to zero. Equations that exhibit fast oscillations, such as those that govern geophysical fluid flow, have posed a problem for the construction of a fast coarse propagator due to the small time steps required to capture the oscillations (even if they are stable for large timesteps, standard methods lose accuracy for timesteps large than the period of the fast oscillations and the PINT algorithm does not converge. Recent work has shown that this difficulty can be overcome by defining a coarse propagator based on asymptotic theory. This propagator incorporates three aspects:

- A heterogenous multiscale timestepping method that is accurate in the highly oscillatory limit, which involves (parallel) averaging of the nonlinear terms over evaluations of the exponential of the linearised wave operator applied to the current state over a number of small timesteps. One large timestep is then taken using the averaged nonlinear terms before iterating.
- A sums-of-rational-approximations method for approximating the exponential of the linearised wave operator; the terms in the sum can be executed in parallel.
- Linear multigrid algorithms for implementing the terms in the sum described above, these can be executed in parallel in space.

The combination with this asymptotically-accurate coarse propagator is called Asymptotic Parallel-in-Time (APINT). It has been demonstrated for a very idealised model (shallow water equations with periodic boundary conditions) [1], but the challenge of application to large scale atmosphere/ocean models is wide open. For three-dimensional baroclinic problems, challenges are: (i) How to efficiently construct the rational approximations for the full 3D baroclinic wave operator? (ii) How to achieve scalable multigrid algorithms for the rational terms? (iii) Is the resulting coarse propagator stable? (iv) Is the resulting coarse propagator accurate enough to lead to PINT convergence? We will concentrate on items (i)-(iii), investigating efficient ways to implement this coarse propagator using the Firedrake software.

Project objectives: We are currently developing a mimetic finite element model for solving the compressible Boussinesq equations in a vertical slice using Firedrake. We will use a linear version of this model and construct a coarse propagator based on rational approximations, based around a general library that could be developed to apply to any wave operator. We will investigate the properties required for accuracy and stability in the Boussinesq system, which possesses much more complicated resonances compared to the shallow water system. The initial coarse propagator code would run in serial since we wish to study accuracy and stability; we will then develop a time-parallel version, using the multiple-parallel-communicator capabilities of PyOP2.

Alignment with PRISM strategy

- *Development of key staff* We have recently secured one year of funding over two years at 0.5FTE for Dr Shipton starting November 2015, and would like to supplement this with another year of funding so that she can return to working full-time (having previously been part-time after returning from maternity leave) with funding until November 2017. This will allow us to seek more subtantial funding in this area, and develop collaborations in the burgeoning APINT community.

- *Collaboration with other PRISM projects* This project will advance the parallel capabilities of Firedrake/PyOP2, particularly the work on multigrid solvers by Dr Lawrence Mitchell. Testbed work on PINT algorithms will allow the PRISM group to determine whether it is a possibility for other application areas.
- *Longer-term research* The goal is to develop initial proof-of-concept results to support substantial long-term funding in this area in collaboration with Beth Wingate (Exeter), Mike Ashworth (STFC Daresbury), working with partners in the atmosphere and ocean community.

References

[1] Terry Haut and Beth Wingate. An asymptotic parallel-in-time method for highly oscillatory PDEs. *SIAM Journal on Scientific Computing*, 36(2):A693–A713, 2014.